- **Vector Line Integrals**
    - $\int_C \vec{F} \cdot d\vec{r}$
    - The vector line integral of a vector field $\vec{F}$ along a piecewise smooth curve C parameterized $\vec{r}(t)$ is $\int_C \vec{F} \cdot d\vec{r} = \int_C \vec{F} \cdot \vec{r}\,'(t)dt$.
        - Note: This is just a substitution: $d\vec{r} = \vec{r}\,'(t)dt$
    - Does depend on orientation!
    - Contrast with scalar line integrals
    - Main application: work along a curve
    - In rare situations, you may encounter a vector line integral $\int_C \vec{F} \times d\vec{r}$, which yields a vector as the solution. Example: Electromagnetism - Biot-Savart Law
- **Vector Surface Integrals in Three-Space**
    - **Flux**
    - $\iint_S \vec{F} \cdot d\vec{S}$
    - The vector surface integral of a vector field $\vec{F}$ through a smooth oriented surface S parameterized $\vec{r}(s,t)$ is $\iint_S \vec{F} \cdot d\vec{S} = \iint_D \vec{F}(\vec{r}(s,t)) \cdot (\vec{r}_s \times \vec{r}_t)dA$, where D is the domain of S in the st-plane.
        - Note: This is just a substitution: $d\vec{S} = \hat{n}d\vec{S} = \frac{\vec{r}_s \times \vec{r}_t}{|\vec{r}_s \times \vec{r}_t|}|\vec{r}_s \times \vec{r}_t|dA = (\vec{r}_s \times \vec{r}_t)dA$
        - Concept of orientable surface: must have two sides. Example: a Möbius strip and a Klein bottle are not orientable (but they are still manifolds).
        - Use right hand rule to determine orientation.
    - Does depend on orientation!
    - You don't need a Jacobian unless using a change of variables
    - Contrast with scalar surface integrals

Further notes:

- **Integrals of a Differential Form on a Manifold in *n*-Space**
    - Manifold must be smooth, parameterized, and oriented.
    - Use a proper change of variables and Jacobian if necessary.
    - Scalar integrals and vector integrals are both integrals of differential forms.